

# CS2020 Fun Quiz!

Herbert

# Warmup

# Warmup

In insertion sort, notice that when you search for the place to insert a new element, you can just binary search the previous elements. For example, in the array:

2 3 5 7 4 1

when inserting 4 to the previous elements, you can just binary search between 2, 3, 5, and 7. Why then does insertion sort not run in  $O(n \log n)$ ?

# Warmup

I have a hash table that contains all CS2020 AY16/17 students' names. Suppose that there are 100 students, and "Le Trung Hieu" is inside this hash table. Which one of these are correct?

- 1 If the hash table has size 10000, it is likely that searching a name takes  $O(1)$  time.
- 2 It is possible that searching for "Le Trung Hieu" fails.
- 3 It is possible that searching for your lovely tutor "Herbert Ilhan Tanujaya" in the hash table succeeds.
- 4 With a hash table of size 200, the hash function that maps a student's name to the total score of the student's problem sets modulo 200 is a good choice of a hash function.

# Warmup

What is the running time of Dijkstra's algorithm, using an unsorted array as the priority queue?

## Special Cases

# Special Cases

Design an algorithm that returns the minimum spanning tree of a connected graph with  $n$  nodes and  $n$  edges in  $O(n)$ .

# Special Cases

You are given an integer array  $A$  with length  $l$ , such that there exists  $k$ ,  $0 \leq k < l$  such that

$$A[k] < A[k + 1] < \dots < A[l] < A[0] < A[1] < \dots < A[k - 1].$$

You don't know what  $k$  is. This is an example of such an array:

4 5 10 12 1 2

Search for an integer in this array in  $O(\log l)$ .



# Special Cases

You are given a connected directed graph  $G$  with edges weighting either 2, 3, or 5. Suppose that  $G$  has  $V$  nodes and  $E$  edges. Design an algorithm that finds the shortest path between two nodes in  $O(E)$  time.

I am the Computer

# I am the Computer

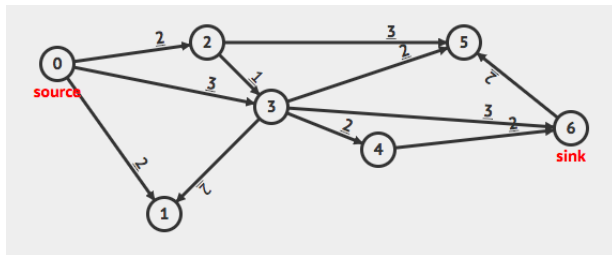
Given below is a skip list. Draw all possible results, with their probabilities, after inserting 9.

## Skip List

2							
2				10			
2	6			10		14	
2	4	6	8	10	12	14	16

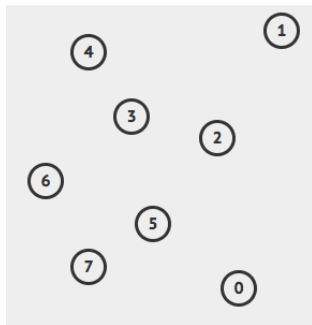
# I am the Computer

Given below is a directed graph. Draw all possible results of the resulting graph after 2 iterations of Ford-Fulkerson.



# I am the Computer

Given below is a set of points. Find the convex hull using the divide-and-conquer algorithm (or otherwise).



# Funky Algorithms

# Funky Algorithms

Consider the following algorithm to find if an element  $k$  exists on a sorted array  $A$ :

*DiceSearch*( $A, k, start, end$ )

- 1 If  $start == end$ , return  $A[start] == k$ .
- 2 Throw a 6-sided fair dice. Let the resulting number be  $d$ .
- 3 Consider

$$new = A \left[ start + \left\lfloor \frac{(end - start)}{d + 1} \right\rfloor \right].$$

If this is  $< k$ , return *DiceSearch*( $A, k, new, end$ ); if  $> k$ , return *DiceSearch*( $A, k, start, new$ ); if  $= k$ , return true.

Will this algorithm return the correct answer? If so, what is the time complexity?

# Funky Algorithms

Consider this hybrid algorithm in finding the minimum spanning tree of a graph:

## Boruvprim's Algorithm

- 1 Do  $O(\log \log V)$  Boruvka steps in the graph
- 2 Collapse the trees found in the Boruvka steps to form another graph.
- 3 Run Prim's algorithm on the resulting graph.

Does this algorithm work? What is the runtime of this algorithm?



# Funky Algorithms

Consider this algorithm that sorts a deck of cards with an integer written on the cards: (assume all integers are distinct)

## Patience Sort (legit name)

- 1 Initially, there are no piles.
- 2 For every card dealt from the deck, place it on the leftmost existing pile, such that the top card's value  $\geq$  the new card's value. If not possible, form a new pile to the right.

For example, the deck of cards in order of  $3 - 7 - 1 - 5 - 6 - 4 - 2$  is dealt into piles of  $(3 - 1)$ ,  $(7 - 5 - 4 - 2)$ ,  $(6)$ .

Why does repeatedly picking the smallest card visible sorts the deck? How do you run this algorithm in  $O(n \log n)$  time?

Lame Stuff

Which tutor relates the most with the operator ++?

# Lame Stuff

Your friend just discovered this amazing new operator `-->` in Java 7 which he calls the “down to” operator! Consider the code snippet below:

## “Down to” Operator usage

```
int x = 10;  
while (x --> 5) System.out.println(x);
```

What is the output?

What is the most used language in programming?

# Big O Analysis

# Big O Analysis

Find the runtime of  $f(n)$ :

## Java snippet

```
void f(int n) {  
    if (n == 1) {  
        System.out.println("hi");  
    } else {  
        for (int i = 0; i < n; i++) {  
            f(n - 1);  
        }  
    }  
}
```

# Big O Analysis

Find the runtime of  $f(n, k)$ :

Java snippet

```
void f(int n, int k) {
    for (int i = 0; i < k * n; i++) {
        System.out.println("hi");
    }
    if (n > 1) {
        f(n / 2, k + 1);
    }
}
```



# Big O Analysis

Find the runtime of  $f(n)$ :

## Java snippet

```
void f(int n) {
    for (int i = 0; i < n; i++) {
        System.out.println(g(i));
    }
}

void g(int n) {
    if (n <= 1) return n;
    else return g(n - 1) + g(n - 2);
}
```

## 2 Java Truths, 1 Java Lie

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Among these three statements, two statements are correct and the other one is incorrect. Find out which one is incorrect and explain all three statements.

- 1 The `String` class is immutable.
- 2 Java's `HashMap` implementation of `containsValue` takes  $O(1)$  time, assuming uniform hashing.
- 3 A static property of a class can be accessed by a non-static method of that class.

## 2 Java Truths, 1 Java Lie

Among these three statements, two statements are correct and the other one is incorrect. Find out which one is incorrect and explain all three statements.

- 1 The % operator in Java can only be used in two integers.
- 2 This results in a compile error:  

```
for(int i=0;i<3;i++) System.out.println(i);  
System.out.println(i);
```
- 3 You can inherit from many interfaces at once.

## 2 Java Truths, 1 Java Lie

Among these three statements, two statements are correct and the other one is incorrect. Find out which one is incorrect and explain all three statements.

- 1 The access level of an overridden method can be changed in a subclass.
- 2 An `int` is passed by value, whereas an `Integer` is passed by reference.
- 3 A Java array can store instances of different classes.

# Finding a Path

# Finding a Path

Design an algorithm that gives you the number of distinct shortest paths from a node  $S$  to the node  $T$  in the connected undirected graph  $G$ .

# Finding a Path

Given a connected undirected graph that models a country's road network, with nodes representing cities and edges representing roads that connect cities. However, each road has a speed limit. Suppose that my car always travel in a constant speed (somehow). Find a path from a city  $S$  to a city  $T$  such that I can travel in the maximum speed possible.



# Finding a Path

Given a connected undirected weighted graph that models a country's road network, with nodes representing cities and edges representing roads that connect cities, with the weights representing the distance of the roads. Suppose that I live in a city  $S$  and my girlfriend lives in a city  $T$ .

Design an algorithm, possibly involving preprocessing, such that I can answer queries of this type efficiently: if there's a road between city  $A$  and city  $B$  with distance  $l$ , will the shortest distance between me and my girlfriend be shorter?

# History

Which one is discovered first - Prim's Algorithm, or Dijkstra's Algorithm?

# History

How many years difference are there between the year MergeSort is discovered, and the year QuickSort is discovered? Your answer must be accurate to 5 years.

What is the thing Dijkstra believed “could only have originated in California”?

Finish Off Remaining Time

# Finish Off Remaining Time

You are given an array with all the numbers from 1 to  $N$  appearing exactly once, except for two numbers missing. Find the two numbers in  $O(N)$  time and constant additional space.

# Finish Off Remaining Time

You have a  $n \times n$  matrix where each cell is either 0 or 1. Find the biggest square matrix in the matrix such that all four borders (edges) contain 1, in  $O(n^3)$  time.



# Finish Off Remaining Time

Design a data structure that stores a collection of integers and supports inserting a number in  $O(\log n)$  time and finding the median in  $O(1)$  time.